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Aeroelastic Divergence of Unrestrained Vehicles

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THE effect of inertial relief on the aeroelastic redistribution of wing lift is discussed in Refs. 1-3. Reference 2 also discusses aeroelastic divergence with a distinction being made between *static* divergence of a restrained vehicle and *dynamic* divergence of an unrestrained vehicle. Equation (7-127) of Ref. 2 states the eigenvalue problem for dynamic divergence of a vehicle with a single rigid-body degree of freedom in plunge. We wish to generalize that result to multiple rigid-body degrees of freedom by following the development of Ref. 3 while using the notation of Refs. 4-6.

The net force distribution $\{F\}$ acting on a flexible lifting surface or body is the difference between the aerodynamic forces $\{F_a\}$ and the inertial forces $\{F_i\}$.

$$\{F\} = \{F_a\} - \{F_i\} \tag{1}$$

The aerodynamic forces have been defined[4,5] in terms of a matrix of steady aerodynamic influence coefficients (AIC's) $[C_{hs}]$ as

$$\{F_a\} = (qS/\bar{c})\,[C_{hs}]\,\{h\} \tag{2}$$

where q is the dynamic pressure, S is the reference area, $\bar{c}$ is the reference chord, and $\{h\}$ is the set of AIC control point deflections. The deflections may be written as the sum of the initial deflections of the rigid vehicle $\{h_r\}$ and the deflections $\{h_m\}$ relative to the mean reference frame (see Refs. 7 and 8)

$$\{h\} = \{h_r\} + \{h_m\} \tag{3}$$

where

$$\{h_m\} = \{h_f\} + [h_R]\,\{a_R\} \tag{4}$$

in which $\{h_f\}$ is the set of flexible deflections relative to the constraint point of the structural influence coefficients (SIC's), $[h_R]$ is the rigid-body modal matrix of Ref. 6, and $\{a_R\}$ is the set of displacements (translations and rotations) of the SIC constraint point relative to the mean reference frame. The requirement for the mean reference frame, when transverse displacement is the primary degree of freedom, is that deformation occurs about it so that the center of gravity does not move and the principal axes do not rotate.[7,8] In terms of the rigid-body modal matrix, this condition is expressed by

$$[h_R]^T[M]\{h_m\} = 0 \tag{5}$$

Equations (4) and (5) lead to

$$\{a_R\} = -[\mathfrak{M}]^{-1}[h_R]^T[M]\{h_f\} \tag{6}$$

where $[\mathfrak{M}]$ is the rigid-body inertial matrix

$$[\mathfrak{M}] = [h_R]^T[M]\,[h_R] \tag{7}$$

and

$$\{h_m\} = [R]\,\{h_f\} \tag{8}$$

where $[R]$ is the inertial relief matrix

$$[R] = [I] - [h_R]\,[\mathfrak{M}]^{-1}[h_R]^T[M] \tag{9}$$

The properties of this idempotent matrix have been discussed by Dugundji[9] and Milne.[10]

The inertial forces are caused by the rigid-body accelerations of the vehicle, which can be found from the mass matrix $[M]$ and the rigid-body modal matrix, and by neglecting structural dynamic response

$$\{F_i\} = [M]\,[h_R]\,\{\ddot{a}_m\} \tag{10}$$

in which $\{\ddot{a}_m\}$ are the accelerations (translational and rotational) of the origin of the mean reference frame. The accelerations are determined from the equilibrium condition for the net forces

$$[h_R]^T\{F\} = [h_R]^T(\{F_a\} - [M]\,[h_R]\,\{\ddot{a}_m\}) = 0 \tag{11}$$

from which

$$\{\ddot{a}_m\} = [\mathfrak{M}]^{-1}[h_R]^T\{F_a\} \tag{12}$$

and

$$\{F\} = [R]^T\{F_a\} \tag{13}$$

The flexible deflections are found from the SIC's and the net forces

$$\{h_f\} = [a]\,\{F\} \tag{14}$$

which may be written in terms of the deflections relative to the mean reference frame and the aerodynamic forces by combining Eqs. (8) and (13) with Eq. (14)

$$\{h_m\} = [R]\,[a]\,[R]^T\{F_a\} = [a_F]\,\{F_a\} \tag{15}$$

where $[a_F]$ has been called the free-body flexibility matrix.[3,11]

Combining Eqs. (2-4) and (15) permits solution for the deflections

$$\{h_m\} = [A]\,[a_F]\,\{F_r\} \tag{16}$$

where $\{F_r\}$ is the aerodynamic force distribution on the rigid system

$$\{F_r\} = (qS/\bar{c})\,[C_{hs}]\,\{h_r\} \tag{17}$$

and $[A]$ is the aeroelastic deflection amplification (or attenuation) matrix

$$[A] = ([I] - (qS/\bar{c})\,[a_F]\,[C_{hs}])^{-1} \tag{18}$$

A singularity in $[A]$ occurs at the eigenvalues of

$$(\bar{c}/Sq)\,\{h_m\} = [a_F]\,[C_{hs}]\,\{h_m\} \tag{19}$$

Aeroelastic divergence corresponds to positive values of $\bar{c}/Sq$. Equation (19) is the desired generalization of Eq. (7-127) of Ref. 2. The eigenvectors of Eq. (19) are the divergence modes which, as deflections from the mean reference frame, are invariant with the selection of the SIC constraint point.

Current interest in swept-forward wings suggests an evaluation of the importance of inertial relief in divergence analysis. An extremely idealized configuration is inset in Fig. 1. The left wing shows the aerodynamic idealization. The wing has an aspect ratio of 4, a taper ratio of 0, and a forward-sweep angle λ. Its span is $4c$, its chord is c, and each semispan is divided into two equal width strips for analysis by strip theory. The right wing shows the structural idealization. The four equal masses on each wing semispan are located along the centerlines of their respective strips and at the one-

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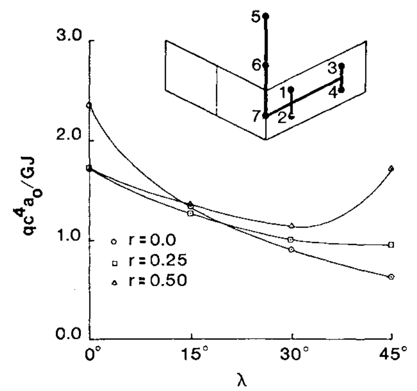

Fig. 1 Variation of the divergence parameter with forward-sweep angle.

quarter and three-quarter chord locations, and are assumed to be connected to the 50% chord elastic axis by rigid streamwise bars. Each wing semispan is assumed to be uniform with equal bending (EI) and torsion (GJ) stiffnesses and connected at its root to the fuselage. The fuselage and trim device (canard) are assumed rigid with three equal and equidistant masses; the fuselage length is $2c$. The mass points are numbered as shown. Aerodynamic forces on the fuselage and canard are neglected, and on the wing are found from subsonic strip theory[4]; the approximation for the strip lift curve slope $c_{l_\alpha} = a_0 \cos \lambda$ is utilized, where a_0 is the airfoil lift curve slope. The solutions of Eq. (19) for the divergence parameter $qc^4 a_0 / GJ$ are also shown in Fig. 1 for $\lambda = 0$ (15) 45 deg and for three weight ratios r (r equals the ratio of wing weight to airplane gross weight) = 0 (the restrained case), 0.25, and 0.50.

For this oversimplified configuration the adverse effect of inertial relief is seen at low-forward-sweep angles, and a favorable effect is seen at the forward limit considered ($\lambda = 45$ deg). [We note that the example vehicle (in the rigid condition) is statically unstable at $\lambda = 45$ deg for the subsonic strip theory assumed (aerodynamic center at 25% of the local chord).] We also note that, again for the example configuration, no significant differences are found between the restrained and unrestrained divergence dynamic pressures for forward-sweep angles between 10 and 25 deg. Certain practical designs have been suggested with forward-sweep angles near 30 deg,[12] and near 45 deg[13] but without consideration of the effects of inertial relief; the influence of inertial relief on those designs remains to be seen. Certainly, the range of sweep angles, both forward and aft, considered by Weisshaar[14-18] and Triplett,[19] while neglecting inertial relief, makes many of their conclusions suspect. Equally certain is that airfoil thickness effects are secondary to inertial relief effects.[20]

Our last question concerns the value of wind tunnel tests in which inertial relief is not simulated.[12,13,21,22] Its simulation would depend on the divergence mode (eigenvector). A typical mode of the example configuration for $\lambda = 30$ deg and $r = 0.25$ has the following normalized mass point deflections (see Fig. 1 for point locations) $h_1 = 0.0849$, $h_2 = -0.0861$, $h_3 = 1.0$, $h_4 = 0.6715$, $h_5 = -0.1269$, $h_6 = -0.1392$, and $h_7 = -0.1515$.† We see a displacement and rotation of the fuselage relative to the mean reference frame; of importance in the simulation is the rotation which, on a flexible fuselage with a canard, would give the canard an angle of incidence varying with dynamic pressure. In the data of Ref. 12 (Table 17), the canard incidence (and its absence) had some effect on the predicted divergence speed. The topic of wind tunnel simulation of inertial relief effects is clearly in need of further study.

†The force equilibrium in the divergence mode illustrates the ensuing motion. For this mode the normalized lift and pitching moment about mass point 7 are $L/h_3 = 0.8653\ a_0 qc$ and $M/h_3 = 0.7946\ a_0 qc^2$, respectively, where $q = 0.9918\ GJ/c^4 a_0$. The translational acceleration of point 7 is $\ddot{z}/h_3 = 0.0520\ a_0 qc/m$ and the pitching acceleration of the fuselage is $\ddot{\theta}/h_3 = 0.00228\ a_0 q/m$ where m is the mass at each wing point. The positive (nose-up) pitching acceleration results from the loss of static stability at this dynamic pressure. This is not exactly the looping maneuver described in Ref. 2; a stable configuration would have a nose-down pitching acceleration, but a neutrally stable configuration would execute the looping maneuver.

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